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On the Application of Optimal Control Theory
to Financial Planning and Forecasting

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Running Title: Financial Planning and Forecasting

Abstract

This study interprets, summarizes, and applies to the financial
planning and forecasting problem a completely general yet operational
approach to the analysis of econometric models by control methods that
has been developed by Gregory Chow. It is shown that the methodology
proposed by Chow not only is well suited for the study of the general
financial planning and forecasting problem facing business firms, but
also suggests a sound approach to the assessment of alternative models.
In addition, this view addresses the problem of the dichotomy between
forecasting models and optimum-seeking models, exposed by Francis
(1983) in an article in this journal.

Key Words

- Financial Planning and Forecasting
- Econometric methods in Finance

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1.   INTRODUCTION

In his review paper on financial planning and forecasting models,
Francis (1983) dichotomizes those models into two categories: fore-
casting models and optimum-seeking models. The first are primarily
designed to forecast but do not attempt to maximize some form of
objective function for the firm. According to Francis, "no optimum
investment or finance theories are included in the typical forecasting
model."[1] Following Francis, optimum-seeking models suffer from the
opposite shortcoming, namely, they fail to include stochastic vari-
ables and thus by and large ignore uncertainty.

Francis found that, surprisingly, little common ground existed be-
tween the two groups of models. He considered this dichotomy analyti-
cally convenient but disturbing nonetheless. He then suggested that
optimal control theory could bridge the gap between those two cate-
gories of financial planning models. However, he left the elaboration
of this approach for future studies.

Beranek (1983), on the other hand, identified several "exploratory"
studies that applied control theory to finance, thus paving the ground
for the "development of analytic solutions to general dynamic financial
planning models."[2] Beranek, however, faults those attempts on two
accounts: their lack of generality and their inability to suit them-
selves to operationalization. Moreover, he observes that although the
assumption of perfect markets invoked by many models is "provocative,"
it is absolutely necessary to recognize it as a special case, the

general case being the existence of varying degrees of imperfections exemplified by transaction costs, the tax structure, and asymmetric information, to name a few.

The purpose of this paper is to follow the path suggested by Francis and explore the application of optimal control theory to financial planning problems, bearing in mind Beranek's observations. This study interprets, summarizes, and applies to the financial planning and forecasting problem a completely general yet operational approach to the analysis of econometric models by control methods that has been developed during about a decade by Chow (1975, 1979, 1981, 1982, 1983). We will show that the methodology proposed by Chow not only is well suited for the study of the general financial planning and forecasting problem facing business firms, but also suggests a sound approach to the assessment of alternative models. In addition, it lends itself to extensions when two or more optimizing agents are involved, e.g., the firm, the financial markets and the government (or regulator).

The paper will develop as follows. Section 2 presents the conceptual framework, with particular emphasis on the translation of theory into econometric modelling using the optimal control approach. Section 3 presents a step-by-step view of the optimal control approach to estimation and optimization, starting with the most general case possible and bringing it down through a linearization process to the linear-quadratic case, which is amenable to estimation. Section 4 discusses the concept of the value function as a standard for comparing and contrasting models in which the optimal control approach is

used. Section 5 presents a critique of the optimal control approach,
with emphasis on the role of assumptions, the tradeoff between effi-
ciency and control, and the problem of institutional factors.
Finally, the last section presents some concluding remarks.

2. Conceptual Framework

The dichotomy between forecasting (i.e., estimation) and optimiza-
tion models becomes less severe when we think of them as parallel
processes, as opposed to extreme ends of a spectrum. This view is
detailed in Exhibit 1.

(INSERT EXHIBIT 1 HERE)

As we can see in Exhibit 1, the processes of estimation (and fore-
casting) and optimization follow a similar structure, starting with
the definition of an objective or loss function and proceeding all the
way down to the analysis of the implications of relaxing the con-
straints. When we look at the processes of estimation and optimiza-
tion in this way, the task of building a bridge between them for the
purposes of financial planning and forecasting appears more feasible
and less daunting than when they are perceived as far apart.

Now consider the basic task of the senior management and the per-
son in charge of building the financial planning model. They need to
devise methods for estimating the parameters of the environment, of
the firm's objective function, and of the resulting behavioral equa-
tions. Underlying the whole process is some sort of maximizing beha-
vior, such as maximizing the stockholders' wealth, which provides
them with behavioral equations, as well as descriptions the economic

environment and the objective function needed in the modelling process. In addition, should the parameters of the economic and financial environment or of the objective function change, the parameters of the behavioral equations would also change. One way of looking into this financial model-building process in the context of an optimal control framework is as follows.[3]

Assume that the firm faces an environment which can be described by a linear system of the form

$$(2.1)$$

$$
\begin{bmatrix} y_{t,1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_{t,p} \end{bmatrix} = \begin{bmatrix} A_{t,11} \text{-----} A_{t,1p} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ A_{t,p1} \quad\quad A_{t,pp} \end{bmatrix} \begin{bmatrix} y_{t-1,1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_{t-1,p} \end{bmatrix} + \begin{bmatrix} C_{t,11} \text{-----} C_{t,1q} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C_{t,p1} \quad\quad C_{t,pq} \end{bmatrix} \begin{bmatrix} x_{t,1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{t,q} \end{bmatrix} + \begin{bmatrix} b_{t,1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ b_{t,p} \end{bmatrix} + \begin{bmatrix} u_{t, \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_{t,} \end{bmatrix}
$$

(px1)   (pxp)   (px1)   (pxq)   (qx1)   (px1)   (p×

or, in matrix notation,

$$y_t = A_t y_{t-1} + C_t x_t + b_t + u_t \qquad\qquad (2.1a)$$

where y_t is a vector of p state variables and x_t is a vector of q control variables; A_t, C_t, and b_t are matrices formed by known constants and u_t is a stochastic disturbance assumed to be serially independent and identically distributed (i.i.d.).

When presented in this form, the financial planning and forecasting problem becomes a control problem. A classical statement of the control problem is due to Intriligator (1971), who states that it consists of

> ...allocating scarce resources among competing ends
> over an interval of time from <u>initial time</u> to
> <u>terminal time</u>. In mathematical terms the problem
> is that of choosing time paths for certain vari-
> ables, called <u>control variables</u>, from a given class
> of time paths called the <u>control set</u>. The choice
> of time paths for the control variables implies,
> via a set of differential equations, called the
> <u>equations of motion</u>, time paths for certain vari-
> ables describing the system, called the <u>state</u>
> <u>variables</u>, and the time paths of the control vari-
> ables are chosen so as to maximize a given func-
> tional depending on the time paths for the control
> and the state variables, called the <u>objective</u>
> <u>functional</u>.[4]

Now, following Chow (1983), assume that the objective function or

loss function[5] which describes the preferences of the firm's decision

makers is quadratic[6]

$$W_t = \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t) \tag{2.2}$$

where K_t is a given symmetric positive semi-definite weighing matrix

and a_t is a vector of targets.

To summarize, the senior management (and the model builder in par-

ticular) is assumed to solve a stochastic optimal control problem,

that is, to find strategies for x_t in order to minimize expected loss

(i.e., maximize expected utility). In other words, they devise their

optimal decision rule for the control variables x_t by minimizing the

expectation of the loss function (2.2) subject to constraint (2.1).

Chow (1983) has demonstrated that, if the model incorporates both

the assumption of a linear environment with an additive disturbance u_t

and that of a quadratic utility function, then the resulting optimal

decision rule has the vector x_t of control variables as a linear

function of the vector y_{t-1} of past endogenous and control variables.
This statement can be translated into a linear feedback control
equation such as

$$x_t = G_t y_{t-1} + g_t, \quad t=1, \ldots, T \tag{2.3}$$

Dynamic programming can be used in order to obtain this optimal deci-
sion rule.[7] Chow (1983:377-8) has shown that the matrices G_t are
obtained by solving the matrix equations

$$G_t = -(C_t' H_t C_t)^{-1} C_t' H_t A_t \tag{2.4}$$

and

$$H_{t-1} = K_{t-1} + (A_t + C_t G_t)' H_t (A_t + C_t G_t) \tag{2.5}$$

backward in time (t=T, T-1, ..., 1) with initial condition $H_t = K_t$.
Furthermore, the vectors g_t are found from the solution of

$$g_t = -(C_t' H_t C_t)^{-1} C_t' (H_t b_t - h_t) \tag{2.6}$$

and

$$h_{t-1} = K_{t-1} a_{t-1} - (A_t + C_t G_t)' (H_t b_t - h_t) \tag{2.7}$$

backward in time (t=T, T-1, ..., 1) with initial condition $h_t = K_T a_T$.

The firm's management can observe time series data on y_t (the
state variables, e.g., short-term borrowing requirements) and x_t (the
control variables, e.g., degree of operating leverage). The parameters
to be estimated by the financial planning and forecasting model are
those of the functions (2.1) to (2.3). However, only the parameters
of (2.1) and (2.2) can be estimated freely; the parameters of (2.3)
are functions of the parameters of the first two equations, insofar as

they are derived by solving the optimal control problem.[8] Moreover, this estimation problem can be solved with the application of the method of maximum likelihood (MLE) subject to the constraints just described on the parameters.

In order to apply this conceptual framework to actual financial planning and forecasting problems, one basic difficulty must be addressed. Equation (2.1) above, which describes the firm's environment, represents a linearized and thus highly simplified view. In general, we should expect that nonlinearities will occur in many, if not most, cases. The central problem, therefore, is to obtain a system like (2.1) from a completely general system of nonlinear simultaneous equations. The following section addresses this question.

3. The Optimal Control Approach to Financial Planning: The General Case

The starting point in the process of building a financial planning and forecasting model expressed as a system of simultaneous nonlinear equations is the realization that the theoretical strengths and weaknesses of a model are better recognized when the model and its properties are investigated as a whole, as opposed to the individual examination of the different equations.

In order to obtain this general assessment of the model, it becomes both desirable and necessary to measure the effects of the control variables on the objective function or expected loss, rather than measuring their effect on any individual endogenous variable. When interpreted in this way, the objective (expected loss) function can be viewed as a composite dependent variable.

This is the general idea which supports the concept of linearization because, in order to be able to make statements about the model's properties, we need first to develop a methodology which essentially translates a completely general financial planning and forecasting model into the linear system (2.1). One method, suggested by Chow (1982), can be summarized as follows.[9]

Chow (1982) starts with a system of nonlinear simultaneous equations written as

$$y_t = \phi(y_t, y_{t-1}, x_t, w_t) + \varepsilon_t \qquad (3.1)$$

where, as before, y_t is a vector formed by state or endogenous variables at time t, and x_t is a vector of control variables. Now Chow (1982) introduces w_t, defined as a vector of exogenous variables not subject to control. In financial planning and forecasting models, typical variables to be included in this vector would be the level of short-term interest rates and the term structure of interest rates. Furthermore, ε_t is defined as a random vector of errors characterized by zero mean and independent distributions through time. Finally, ϕ is a vector of functions ϕ_i which includes nonlinear functions in general but also linear functions in particular.

Since Chow (1982, 1983) defines the vector of control variables x_t as a subvector of y_t, he is able to use the objective or loss function (2.2),

$$W = \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t) = \sum_{t=1}^{T} (y_t' K_t y_t - 2y_t' K_t a_t + a_t' K_t a_t) \qquad (3.2)$$

As described in the previous section, the control problem is solved by minimizing expected loss (min E[W]), subject to the set of restrictions represented by the system (3.1).

Now we come to the crucial point that, in the view taken in this work, makes Chow's approach to estimation by control methods amenable to be used as a paradigm for financial planning models which attempt to bridge the gap between estimation and forecasting, on one side, and optimization, on the other. Chow was able to devise a simple series of steps which reduce the completely general problem represented by (3.1)-(3.2) to the linear-quadratic case of the previous section. In other words, this means that in building financial planning models, restrictive assumptions on the environment which lead to linear relationships need not be imposed at the outset. This addresses both of Beranek's criticisms discussed above.

Chow's step-by-step linearizing solution can be summarized as follows. The first step is to identify a "tentative policy path" represented by x_1^0, x_2^0, ..., x_T^0, given w_1, w_2, ..., w_T. In a typical financial planning model, this may include, say, choosing the degree of financial leverage over time, given the term structure of interest rates. By imposing some additional conditions, namely $\varepsilon_t = 0$ for all T periods, it is possible to find a solution path y_1^0, y_2^0, ..., y_T^0 for the vector of state variables.

The system of simultaneous nonlinear equations (3.1) is then linearized about the point y_t^0, y_{t-1}^0, and x_t^0, which leads to the system of <u>linear</u> simultaneous equations[10]

$$y_t = y_t^0 + B_{1t}(y_t - y_t^0) + B_{2t}(y_{t-1} - y_{t-1}^0) + B_{3t}(x_t - x_t^0) + \varepsilon_t \qquad (3.3).$$

Chow explains that the elements of the matrices B_{1t}, B_{2t}, and B_{3t} can be found by differentiating the functions ϕ_1 with respect to be elements of y_t, y_{t-1}, and x_t.

The solution to the system of linear simultaneous equations (3.3) then leads to the linearized reduced-form expression below, which is formally identical to (2.1):

$$y_t = A_t y_{t-1} + C_t x_t + b_t + u_t \qquad (3.4)$$

where

$$(A_t; C_t; u_t) = (I - B_{1t})^{-1}(B_{2t}; B_{3t}; \varepsilon_t) \qquad (3.5)$$

$$b_t = y_t^0 - A_t y_{t-1}^0 - C_t x_t^0.$$

The final step in Chow's operational procedure is to use the linear description of the environment and the loss function to obtain the optimal linear feedback control expression

$$\hat{x}_t = G_t y_{t-1} + g_t \qquad (3.6)$$

Note, however, that (3.6) differs from (2.3) because $\hat{x}_t$ results from the tentative policy path x_1^0, x_2^0, ..., x_t^0 chosen in the first step. The remainder of Chow's linearizing procedure consists of choosing new tentative policy paths and following the same sequence of steps. Eventually, he arrives at a stochastic system under control from which he can obtain the mean path and covariance matrix. The successive iterations, of course, are obtained with the use of a computer program.[11]

The complete generality of this approach should be apparent. It
is not necessary to impose restrictive assumptions at the outset on
either functional relationships internal to the firm or those existing
in the firm's environment. In particular, it is not necessary to
brush aside financial market imperfections. Therefore, the applica-
tion of Chow's procedure to financial planning and forecasting is not
only theoretically sound but feasible.

Moreover, one remaining difficulty with existing financial planning
models is that, because of their more or less particular properties,
it becomes difficult to compare them. With the introduction of Chow's
methodology, different models can now be compared with the use of the
so-called "value function." This is discussed in the next section.

4. <u>Comparing and Contrasting Alternative Models: The Value Function</u>[12]

The operational solution of the financial planning and forecasting
problem stated as a control problem involves minimizing the sum of the
expected losses in (3.2) for all T periods (i.e., the financial
planning horizon) with respect to the vector of control variables x_1
of the first period, assuming the optimality of all subsequent control
vectors x_2, ..., x_T. This can be accomplished with the use of dynamic
programming which, by a process of backward optimization in time,
seeks to find the optimal feedback control equations for all t periods,
starting with the last period T, given the loss function (3.2) and the
linearized, reduced-form expression (3.4).

Chow (1981, 1983) explains that this summation, which takes into
account all the future minimum expected losses from period 2 to the
final period T, then can be written as the expectation of a quadratic

function expressed in terms of the vector of state variables for the

first period y_1, and known matrices H_1 and c_1,

$$V_1 = E(y_1'H_1y_1 - 2y_1'h_1 + c_1) \tag{4.1}$$

where the elements of H_1 and h_1 are found by applying (2.5) and (2.7)

to the initial period and c_1, as shown in Chow (1975), is obtained by

solving[13]

$$c_{T-1} = a_{T-1}'K_{T-1}a_{T-1} + (b_T + C_T g_T)'H_T(b_T + C_T g_T)$$

$$- 2(b_T + C_T g_T)'h_T + c_T + E_{T-1}u_T'H_T u_T. \tag{4.2}$$

with T=2.

The expectation obtained in (4.1) is the so-called "value function,"

which Chow proposes as a paradigm for comparing econometric models

estimated by control methods, thus performing a similar task to that

of the likelihood function in more traditional methods. The impor-

tance of this concept when applying this estimation approach to finan-

cial planning and forecasting problems cannot be overemphasized. For

one of the major difficulties in comparing existing models, forecasting

and optimum-seeking models alike, is that their particular features

prevent a unique standard of evaluation from being applied. The value

function overcomes this difficulty.

However, the value function, in order to be operational, must be

written in terms of the initial control vector x_1. The reason is that

the minimization of V_1 with respect to x_1 produces $\hat{x}_1$, which is the

optimal initial-period control vector (or policy). Then, by imposing

the additional assumption that optimal control vectors (i.e., policies) will be found for all subsequent periods (i.e., from period 2 to T), the expected multiperiod loss becomes also a function of x_1. Finally, the <u>minimum expected multiperiod loss</u> $\hat{V}_1$, which will be the ultimate criterion for model selection, becomes a function of $\hat{x}_1$, thus $\hat{V}_1 = V_1(\hat{x}_1)$. Chow (1982) shows that this is accomplished by substituting (3.4) for y_1 in (4.1) and taking expectations to obtain

$$V_1 = x_1'Qx_1 + 2x_1'q + d, \tag{4.3}$$

where

$$Q = C_1'HC_1, \tag{4.4}$$

$$q = C_1'(H_1A_1y_0 + H_1b_1 - h_1), \tag{4.5}$$

and

$$d = c_1 + (A_1y_0 + b_1)'H_1(A_1y_0 + b_1) + Eu_1'H_1u_1 - 2(A_1y_0 + b_1)'h_1. \tag{4.6}$$

In Chow's own words:

> This [value] function gives the total expected loss from period 1 to period T in terms of the control variables x_1 in the first period, assuming that future policies from period 2 to period T shall be optimally chosen. It appears to capture the essential information contained in an econometric model concerning the effects of the current policy variables on economic welfare as measured by the loss function.[14]

Notice that by substituting "financial planning and forecasting" for "econometric" and "the value of the firm" for "economic welfare"

we obtain a working definition of this standard which suits well the problem under study.

The optimal control approach to financial planning and forecasting, therefore, appears to be superior to conventional models of either the forecasting or the optimum seeking variety for two reasons. First, because it overcomes the existing dichotomy between forecasting and optimum-seeking models. Secondly, because it lends itself to a unique and completely general standard for model evaluation and comparison. This approach, however, is not immune from criticism and has its own limitations, which are discussed in the following section.

5. Limitations of the Optimal Control Approach to Financial Planning and Forecasting

This critique will revolve around three topics, namely, the role of assumptions in the model, the tradeoff between efficiency and control, and the problem of institutional factors.

With respect to the first topic, it is important to realize that the optimal control approach assumes that: (a) the economic agents (i.e., the firm and its environment) and the model builder agree with respect to the description of the environment in (2.1); (b) the description of the environment itself is correct; and (c) the objective or loss function (2.2) has been correctly specified.

Yet even if we believe in those assumptions, the optimal policy rule (2.3) will <u>not</u> be truly optimal because, as Chow (1983) notes, neither the firm (i.e., the model builder) nor the institutions comprising the environment will know (and cannot be assumed to know) the exact numerical values of the elements of the matrices A, C, and b.

The application of the existing methods for computing (2.3) yields only the so-called uncertainty-equivalent solution, which is not optimal when A, C, and b have uncertain components.

In addition, the process of learning and of updating environmental information (i.e., [2.1]) may not be taken into account explicitly and may even be ignored by model-builders.[15] Indeed, Chow (1983) observes that this seems to be the rule rather than the exception, insofar as "[model-builders] assume ...that a steady state is always observed for the optimal behavioral equation (2.3) ...[thus] the question is how far one should push optimizing behavior in building economic models for multiperiod decision under uncertainty and where on should stop."[16]

The second topic under consideration deals with efficiency, which is generally considered a desirable property for an optimal rule or policy. Anderson and Taylor (1976) and Taylor (1976), among others, studied this problem. Their conclusions should add a tone of caution when analyzing the implications of using the optimal control method on the estimates obtained from the model, i.e., the optimal path for the control variables.

Taylor (1976) worked with the case where the dependent variable is being controlled by all the independent variables. He shows that some single period optimal control rules yield unacceptable parameter estimates. He then proposed a class of (parameter-estimating) certainty equivalence rules as an alternative, and showed that they obtain much greater sampling efficiency with small loss in control efficiency. However, he also found that the more parameters that are added, the lower is the control efficiency, suggested a <u>tradeoff between estimation</u>

and control.[17] This is a potential problem in financial planning and forecasting models, where the number of parameters tends to be large.

The results of Anderson and Taylor (1976) pointed to the same direction. They were able to show that the estimator proposed above, namely, the least squares certainty equivalence rule (LSCE) is asymptotically consistent; on the other hand, while a linear combination of the parameter estimates was shown to be consistent, their data failed to confirm consistency for the individual estimates.[18] Along the same lines, Fair (1974) tempered the generally optimistic tone of his paper by noting that "...there is an obvious trade off between the size of the model, the number of control variables, and the length of the decision horizon. It is hard to establish any precise rules as to what problems are practical to solve and what are not because no two models and problems are the same."[19]

Finally, the builder of a financial planning and forecasting model will be ill-advised if she ignores institutional factors, whether or not the optimal control approach is used. This attitude may render the model meaningless, no matter how powerful is the estimation technique applied. The upshot is that, if the model-builder chooses to ignore some fundamental institutional constraints present in the environment for the sake of being able to reduce the modelling task to manageable portions,[20] she must be willing to accept the fact that the resulting policy rule, conclusions or recommendations will have to be looked at with caution.

Some final observations are presented in the next section, which closes this study.

6. <u>Concluding Remarks</u>

This study has investigated the optimal control approach to financial planning and forecasting, following the suggestion of Francis (1983) and keeping in mind the criticisms of Beranek (1983). It has been shown that not only the optimal control method is more suitable to the financial planning problem, for it is dynamic in nature, but it is also a completely general approach which does not require the imposition of constraints on the environment at the outset. In particular, the assumption of perfect markets does not need to be invoked.

This work has proposed that when the structure of optimization and forecasting models is seen as parallel, as opposed to characterizing those models as opposite extremes of a spectrum, the task of building a bridge between them becomes more feasible. Following the thorough, step-by-step methodology suggested by Gregory Chow in a series of econometric studies, it was shown that even the most general model possible can be operationalized into the linear-quadratic case, which lends itself readily to estimation and for which computing algorithms already exist. The concept of the value function, also developed by Chow, has been proposed as a standard for comparing and contrasting alternative financial planning and forecasting models which use this approach. Finally, a critique of this approach has been presented, for the understanding of its limitations is crucial to the task of model-building.

Overall, the optimal control approach to financial planning and forecasting appears sound and compares favorably to existing approaches, insofar as it brings to this estimation and forecasting

problem the basic assumption of firm maximizing behavior which forms

the core of microfinance. This paper, therefore, should close with a

note of optimism. Mindful that a good technique cannot be a substitute

for bad theory, the model builder should be careful enough and take

into account the limitations discussed here, and others that she may

face in the particular problem under study. However, she will be

better off by using a methodology which ultimately recognizes that

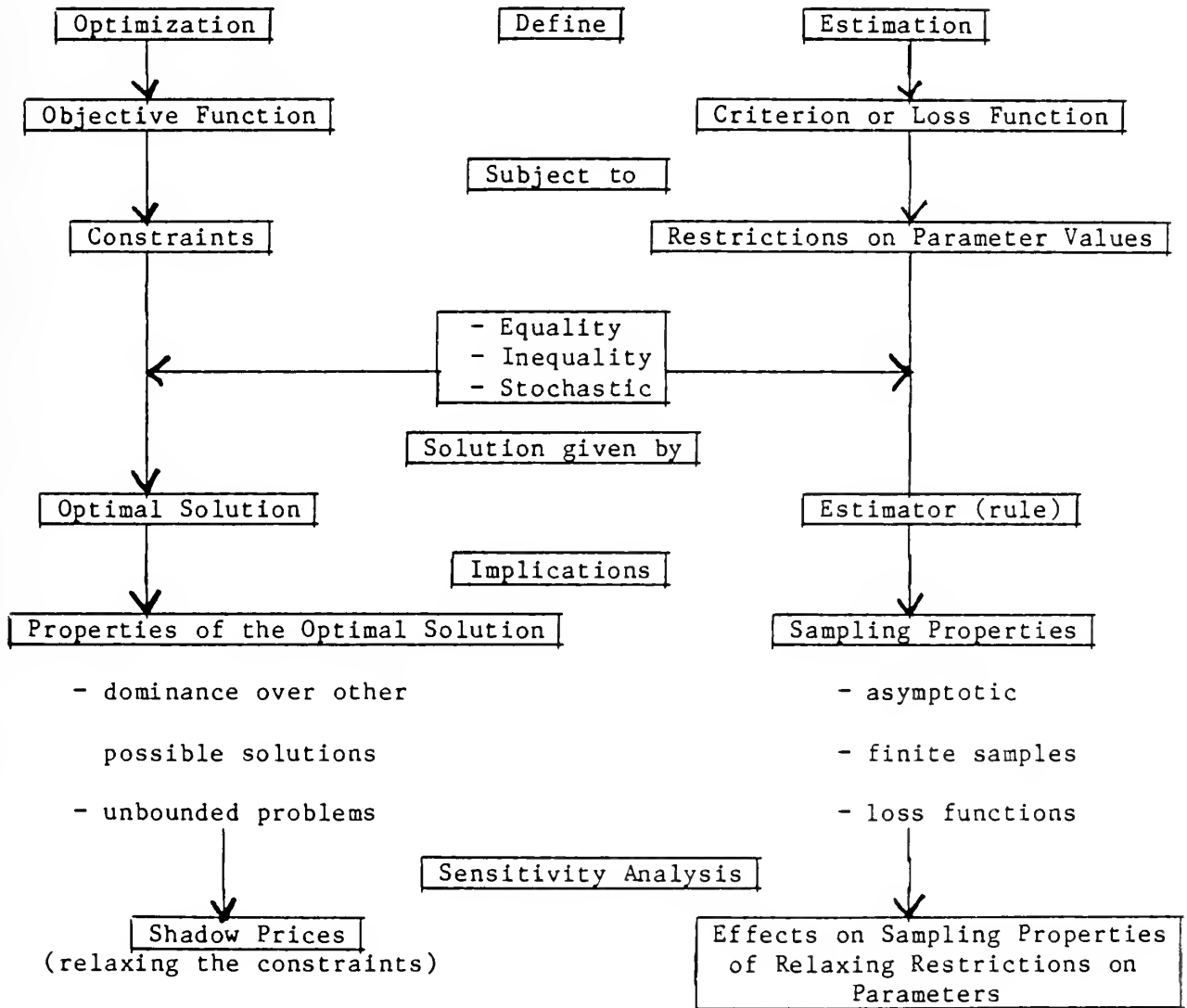firms attempt to optimize over time.

ACKNOWLEDGMENT

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EXHIBIT 1

Parallels Between the Processes of Estimation and Optimization

| Optimization | Define | Estimation |

Objective Function → Criterion or Loss Function

Subject to

Constraints → Restrictions on Parameter Values

- Equality
- Inequality
- Stochastic

Solution given by

Optimal Solution → Estimator (rule)

Implications

Properties of the Optimal Solution → Sampling Properties

- dominance over other
  possible solutions
- unbounded problems

- asymptotic
- finite samples
- loss functions

Sensitivity Analysis

Shadow Prices
(relaxing the constraints)

Effects on Sampling Properties
of Relaxing Restrictions on
Parameters

.

NOTES

[1] Francis (1983), p. 285.

[2] Beranek (1983), p. 717.

[3] The following discussion relies heavily on Chow (1983), esp. Chapter 12, pp. 377-380. A thorough treatment is presented there.

[4] Intriligator (1971), p. 292.

[5] Recall that loss minimization and utility maximization are equivalent concepts.

[6] As pointed out by Chow (1983), p. 377, the loss function (eq. 2.2) does not need to have x_t as an argument because the state variables have been introduced in such form so as to incorporate x_t as a subvector of y_t.

[7] The specifics of the method of dynamic programming have been described in detail elsewhere. See, for example, Chow (1982), p. 153.

[8] Chow (1983) makes this point.

[9] A more comprehensive description of this treatment, including instructions as to how to obtain a computer program that performs the calculations, can be found in Chow (1982), pp. 150-1.

[10] Chow (1982), p. 150.

[11] Chow (1982), p. 151.

[12] The concept of the value function is developed by Chow in at least Chow (1975), Chapter 8, and Chow (1982), pp. 152-4. This section relies primarily on the latter.

[13] See Chow (1975), p. 179.

[14] Chow (1982), p. 153, insertion mine.

[15] Note, however, the sheer complexity which could result from this process of continuously updating the environmental description (2.1). This is outside the scope of this paper, but it does seem worthwhile as a topic for future study.

[16] Chow (1983), p. 395, insertion mine.

[17] Taylor (1976), pp. 339, 346-347.

[18] Anderson and Taylor, pp. 1289-90, 1302.

[19]Fair (1974), pp. 135, 149.

[20]Of course no model can include all institutional constraints, not even a significant part of those. In addition, there is no "right" measure of attention to be given to the institutional factors present in the environment. What is being argued here is that a good modelling approach is not a substitute for bad judgement. Ultimately, the financial planning and forecasting model will reflect its builder's abilities.